Aeronautical Eng. Department **Level: 3rd Year** **Examiner: Dr. Mohamed Eid** **Time allowed: 3 hours**	Institute of Aviation Engineering and Technology	**Semester: Autumn 2018** **Final Exam** **Course: Mathematics IV** **Code: Math 301** **Date: January 2, 2019**

The Exam consists of one page **Answer all questions** **No. of questions: 4** **Total Mark:** 55

Question 1

(a)Prove that : $B(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$ 4

(b)Find the integrals: (i) $\int_1^\infty e^{-(x-1)^2}\,dx$ (ii) $\int_0^\infty \frac{1}{1+x^4}\,dx$ 6

Question 2

(a)Solve the PDE: (i) $2u_{xx} - 5u_{xy} + 2u_{yy} = e^{2x+3y}$ (ii) $9u_{xx} - 4u_{yy} = \cos(2x)$ 8

(b)Solve the wave equation: $u_{tt} - 16u_{xx} = 0, \quad 0 < x < 1$

$\text{B.C. } u(0,t) = u(1,t) = 0 \text{ and I.C. } u(x,0) = x, \quad u_t(x,0) = 1$ 4

(c)Solve the heat equation: $u_t - 4u_{xx} = 0, \quad 0 < x < 1.$

$\text{B.C. } u(0,t) = u(1,t) = 0 \text{ and I.C. } u(x,0) = x + 1.$ 4

Question 3

(a)From the data: (1, 2), (2, 5), (3, 6), (4, 10), (5, 12) 6

Find the regression line $y = a + bx$, $\bar{x}$, $\bar{y}$, σ_x, σ_y and the correlation coefficient r.

(b) If x is random variable with pdf $f(x) = e^{-x}$, $x \geq 0$. Find the moment generating function $M_x(t)$ and from it, find m_0, m_1, m_2, V *and* σ. 6

(c)If x, y are random variables with pdf $f(x,y) = \frac{3}{2}x^2 + y$, $0 < x, y < 1$. Find $\text{cov}(x,y)$ 4

Question 4

(a)Find the probabilities $P(x = 3)$, $P(x \leq 5)$, $P(x < 5)$, $P(x > 4)$ from the data: 3

x_i	$x < 2$	2	3	4	5	6	$x > 6$
$f(x_i)$	0	0.1	0.2	0.3	0.1	0.3	0

(b)Four fair coins are tossed simultaneously and x is the number of appearing heads H. 3
From the binomial distribution, find the probabilities : $P(x = 2)$, $P(x < 3)$, $P(x \geq 3)$

(c)In a production of iron rods, if the diameter x is normally distributed with $\mu = 2$ and $\sigma = 0.008$. Find the percentage of non defectives with tolerance limits 2 ± 0.02 that is, find $P(1.98 \leq x \leq 2.02)$ where $\phi(2.5) = 0.9938$ 3

(d)From the Gamma distribution: $f(x) = \begin{cases} \frac{1}{\Gamma(n)} x^{n-1} e^{-x}, & x, n > 0 \\ 0, & Otherwise \end{cases}$ 4

Show that $\sigma = \sqrt{n}$. Also, find $P(x \leq 2)$ and $P(x > 2)$ when n = 2.

Good Luck *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a)Proof : $B(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$

---4 Marks

(b)(i)Put $y = (x-1)^2$, $dy = 2(x-1)dx$ and $dx = \frac{1}{2\sqrt{y}}dy$

Then $I = \frac{1}{2}\int_0^\infty y^{-\frac{1}{2}}e^{-y}\,dy = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}$

(b)(ii)Put $y = x^4$, $dy = 4x^3dx$ and $dx = \frac{1}{4}\frac{1}{y^{\frac{3}{4}}}dy$

Then $I = \frac{1}{4}\int_0^\infty \frac{y^{-\frac{3}{4}}}{1+y}dy = \frac{1}{4}B(\frac{1}{4},\frac{3}{4}) = \frac{\pi}{4}\sqrt{2}$

---6 Marks

Answer of Question 2

(a)(i)A.E $2k^2 - 5k + 2 = 0$, then $k = 2,\ \frac{1}{2}$ and $y_c = f(y+2x) + g(y+\frac{1}{2}x)$

$$y_p = \frac{1}{2D^2 - 5DE + 2E^2}e^{2x+3y} = \frac{1}{8-30+18}e^{2x+3y} = -\frac{1}{4}e^{2x+3y}$$

(a)(ii)A.E $9k^2 - 4 = 0$, then $k = \pm\frac{2}{3}$ and $y_c = f\left(y+\frac{2}{3}x\right) + g\left(y-\frac{2}{3}x\right)$

$$y_p = \frac{1}{9D^2 - 4E^2}\cos 2x = \frac{1}{-36-0}\cos 2x = -\frac{1}{36}\cos 2x$$

---8 Marks

(b)Wave equation $c = 4$, $L = 1$, $f(x) = x$, $g(x) = 1$. Then

$$A_n = \frac{2}{1}\int_0^1 x.\sin n\pi x\,dx = 2[\frac{-x}{n\pi}\cos n\pi x + \frac{1}{(n\pi)^2}\sin n\pi x] = \frac{2}{n\pi}(-1)^{n+1}$$

$$B_n = \frac{2}{4n\pi}\int_0^1 1.\sin n\pi x\,dx = \frac{-1}{2n\pi}\frac{\cos n\pi x}{n\pi} = \frac{-1}{2(n\pi)^2}[(-1)^n - 1] = \begin{cases}\frac{1}{(n\pi)^2}, & n \text{ is odd}\\ 0, & n \text{ is even}\end{cases}$$

$$U(x,t) = \sum_{n=1}^{\infty}\frac{2}{n\pi}(-1)^{n+1}\sin n\pi x\cos 4n\pi t + \sum_{n=1}^{\infty}\frac{1}{\pi^2(2n-1)^2}\sin(2n-1)\pi x.\cos 4(2n-1)\pi t$$

---4 Marks (c)Heat equation

$$B_n = \frac{2}{1}\int_0^1 (x+1)\sin n\pi x \, dx = \frac{2}{n\pi}[1-2(-1)^n]$$

$$U(x,t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}[1-2(-1)^n] \sin n\pi x \,.\, e^{-t(2n\pi)^2}$$

--4 Marks

Answer of Question 3

(a)By regular calculator, the regression line is $y = a + bx = -0.5 + 2.5x$, $\bar{x} = 3$, $\bar{y} = 7$, $\sigma_x = 1.414$, $\sigma_y = 3.578$ and the correlation coefficient $r = 0.988$

--6 Marks

(b) $M_x(t) = \int_{-\infty}^{\infty} e^{xt} f(x)\, dx = \int_0^{\infty} e^{xt}\, e^{-x} dx = \int_0^{\infty} e^{xt}\, e^{-(1-t)x} dx = \frac{-1}{1-t} e^{-(1-t)x} = \frac{1}{1-t}$

By the binomial theorem, we get

$$\frac{1}{1-t} = 1 + t + t^2 + t^3 + \cdots$$

$$= 1 + t + 2\frac{t^2}{2!} + 6\frac{t^3}{3!} + \cdots$$

Then $m_0 = 1$, $m_1 = 1$, $m_2 = 2$, $m_3 = 6$, $V = 2 - 1 = 1$ *and* $\sigma = 1$

--6 Marks

(c) $E(x) = \int_0^1 \int_0^1 x\ f(x,y) dx\, dy = \int_0^1 \int_0^1 x \left(\frac{3}{2}x^2 + y\right) dx\, dy = \frac{5}{8}$

$$E(y) = \int_0^1 \int_0^1 y\ f(x,y) dx\, dy = \int_0^1 \int_0^1 y \left(\frac{3}{2}x^2 + y\right) dx\, dy = \frac{7}{12}$$

$$E(xy) = \int_0^1 \int_0^1 x\, y\, f(x,y) dx\, dy = \int_0^1 \int_0^1 x\, y \left(\frac{3}{2}x^2 + y\right) dx\, dy = \frac{17}{48}$$

Then $cov(x,y) = E(xy) - E(x)E(y) = -\frac{1}{96}$

--4 Marks

Answer of Question 4

(a) $P(x = 3) = 0.2,\ \ P(x \le 5) = 0.7,\ \ P(x < 5) = 0.6,\ \ P(x > 4) = 0.4$

--3 Marks

(b) $n = 4,\ \ P(x = 2) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

$P(x < 3) = f(0) + f(1) + f(2) = \binom{4}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 + \binom{4}{1}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 + \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$

$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16} = \frac{11}{16}$

$P(x \ge 3) = 1 - P(x < 3) = \frac{5}{16}$

--3 Marks

$(c)\ Since\ \ P(a \le x \le b) = F(b) - F(a) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$

Then $P(1.98 \le x \le 2.02) = \phi\left(\frac{2.02-2}{0.008}\right) - \phi\left(\frac{1.98-2}{0.008}\right)$

$= \phi(2.5) - \phi(-2.5) = \phi(2.5) - [1 - \phi(2.5)]$

$= 0.9938 - [1 - 0.9938] = 0.9876 = 98.76\ \%$

--3 Marks

$(d)\ E(x) = \mu = \frac{1}{\Gamma(n)}\int_0^\infty x.\, x^{n-1}e^{-x}\, dx = \frac{\Gamma(n+1)}{\Gamma(n)} = n$

$E(x^2) = \frac{1}{\Gamma(n)}\int_0^\infty x^2.\, x^{n-1}e^{-x}\, dx = \frac{\Gamma(n+2)}{\Gamma(n)} = n(n+1)$

$V = E(x^2) - (\mu)^2 = n(n+1) - n^2 = n \quad and \quad \sigma = \sqrt{n}$

When $n = 2,$ then $P(x \le 2) = \frac{1}{\Gamma(2)}\int_0^2 x\, e^{-x}\, dx = 0.59$

$P(x > 2) = 1 - P(x \le 2) = 0.41$

--4 Marks

Dr. Mohamed Eid

Autumn 2018 | Math IV | Mid-Term Exam

Communications & Aviation | Time: 1 Hour | Total Mark : 15

(1)Write the general form of Gamma function and then find $\Gamma(-\frac{1}{2})$.

(2)Write the general form of Beta function and then find $B(-\frac{1}{2},\frac{3}{2})$.

(3)Find the integrals : (a) $\int_3^\infty e^{-(x-3)^2}\,dx$ (b) $\int_{-\infty}^\infty \frac{e^x}{(1+e^{2x})^2}\,dx$

(4)Solve the PDE :

(a) $u_{xx} - 4u_{xy} + 3u_{yy} = e^{2x-y}$ (b) $u_{xx} - 4u_{yy} = x^3 + y^2$

(c) $u_x + u_y = xy$

Good Luck *Dr. Mohamed Eid*